

## AN EFFICIENT COMPUTATIONAL MODEL FOR CO<sub>2</sub> FLOW IN POROUS MEDIA

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**Abstract.** This paper presents an efficient computational model for the simulation of fully coupled two-phase flow in a deformed partially saturated formation. Focus is placed on modeling the flow of a CO<sub>2</sub> plume in a porous medium. The numerical procedure is based on coupling between the Level-set method (LS) and the extended finite element method (XFEM). The level set is employed to define the location of the front between injected CO<sub>2</sub> and existing formation water. A streamline upwind Petrov-Galerkin method is utilized to stabilize the possible occurrence of spurious oscillations in the advection of the CO<sub>2</sub> front. The XFEM is employed to model the gradient in the degree of saturation at the front. This is done by decomposing the saturation field into a continuous part and a discontinuous part. The latter is enhanced by the use of a local enrichment function which is calculated on the basis of the Level-set function. Numerical implementation of the method is discussed and numerical examples are given.

### 1 INTRODUCTION

Carbon dioxide sequestration is considered to be a promising method for reducing the emission of CO<sub>2</sub> into the atmosphere. This process can be achieved by capturing CO<sub>2</sub> from sources, e.g. flue gases in power plants, and then injecting it into a variety of underground formation, including active or depleted oil and gas reservoirs, subsurface aquifers and coal beds.

The injection of CO<sub>2</sub> into underground formation involves coupled multi-physical processes including fluid flow in porous media and hydro-mechanical processes. The simulation of CO<sub>2</sub> flow and its propagation in porous media is essential for understanding these processes, designing field parameters and monitoring the process.

To attain a high level of accuracy and a stable result for the convection-dominated flow process especially in a large domain, conventional simulators require a large number of finite elements and significant CPU time.

In this work the level set method and the partition of unity method are used to model CO<sub>2</sub> injection in a saline aquifer. The level set method is used to capture the interface between CO<sub>2</sub> and water and the evolution of CO<sub>2</sub> plumes during injection. A streamline upwind Petrov-Galerkin method is utilized to stabilize the possible occurrence of spurious oscillations in the advection of the CO<sub>2</sub> front. The partition of unity finite element method is employed to model the sharp front within the element. This is done by decomposing the saturation field into a continuous part and a discontinuous part. The latter is enhanced by the use of a local enrichment function which is calculated on the basis of the level set function.

The Level set method was first introduced by Osher and Sethian [8] for capturing moving fronts. This method makes use of a distance function, referred to as the level-set function, which labels every point with a sign and a value. The sign indicates the fluid domain and the value represents the shortest distance to the interface. As such this function is equal to zero at the interface and non-zero elsewhere. The interface is advected with a local flow velocity, using any of the known advection equations.

However, when using the standard Galerkin finite element method to solve the LS equation, the advection term creates a skew matrix which is the source of non-physical oscillations. One of the most popular solutions of this problem is known as the Streamline Upwind Petrov-Galerkin method (SUPG). It was proposed by Hughes and Brooks in [3]. In SUPG method the shape function is perturbed in the direction of the flow. There is a vast amount of literature regarding the design and amount of this perturbation. The formulation presented in [11] is used in this study.

In XFEM the standard finite element space is enriched with special functions to capture the discontinuity or high gradient in the problem. The method was first utilized for the simulations of cracks in structures without the need of re-meshing (see for example Belytschko and Black [1] and Moes et al [6]). Sukumar et al. [9] were the first to combine the XFEM with a level set method to make the enrichment function related to interfaces.

In this paper we present some preliminary results of the proposed model.

## 2 GOVERNING EQUATIONS

The physical model which is used in this approach is the same as that presented by Lewis and Schrefler [5] for deforming multiphase flow in porous media. The macroscopic field variables are displacement, wetting fluid pressure and non-wetting saturation. The governing equations of the coupled multiphase flow are briefly presented hereafter.

### 2.1 Equilibrium equations

For a three-phase medium, i.e. solid, water and CO2 under static loading, the linear momentum equation in terms of total stress tensor,  $\boldsymbol{\sigma}$  can be presented as:

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} = 0 \quad (1)$$

where  $\mathbf{g}$  is the gravity force vector per unit mass and  $\rho$  is the density given by:

$$\rho = (1-n)\rho_s + n S_w \rho_w + n S_g \rho_g \quad (2)$$

in which  $\rho_s$  is the solid density,  $\rho_w$  is the water density,  $\rho_g$  is the CO2 density,  $S_w$  is water saturation,  $S_g$  is CO2 saturation and  $n$  is the porosity.

The effective stress principle used is given by:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}'' - \mathbf{m}^T \alpha (S_w P_w + S_g P_g) \quad (3)$$

in which,  $\boldsymbol{\sigma}''$  is a general form of the effective stresses,  $P_w$  and  $P_g$  are water and gas pressure,  $\alpha$  is the Biot constant and  $\mathbf{m}^T = [1, 1, 1, 0, 0, 0]^T$ .

## 2.2 Continuity Equations

The continuity equation for the water phase and the gas phase can be respectively expressed as:

$$\left( \frac{\alpha-n}{K_s} S_w^2 + \frac{nS_w}{K_w} \right) \frac{\partial P_w}{\partial t} + \frac{\alpha-n}{K_s} S_w S_g \frac{\partial P_g}{\partial t} + \alpha S_w \mathbf{m}^T \mathbf{L} \frac{\partial \mathbf{u}}{\partial t} + \left( \frac{\alpha-n}{K_s} P_w S_w - \frac{\alpha-n}{K_s} P_g S_w + n \right) \frac{\partial S_w}{\partial t} + \nabla^T \left[ \frac{\mathbf{k} k_{rw}}{\mu_w} (-\nabla P_w + \rho_w \mathbf{g}) \right] = 0 \quad (4)$$

$$\frac{\alpha-n}{K_s} S_w S_g \frac{\partial P_w}{\partial t} + \frac{\alpha-n}{K_s} S_g^2 \frac{\partial P_g}{\partial t} - \left( \frac{\alpha-n}{K_s} S_g (P_g - P_w) + n \right) \frac{\partial S_w}{\partial t} + \alpha S_g \mathbf{m}^T \mathbf{L} \frac{\partial \mathbf{u}}{\partial t} + \frac{nS_g}{\rho_g} \frac{\partial}{\partial t} \left( \frac{P_g M_g}{\theta R} \right) + \nabla^T \left[ \frac{\mathbf{k} k_{rg}}{\mu_g} (-\nabla P_g + \rho_g \mathbf{g}) \right] = 0 \quad (5)$$

in which  $K_s$  and  $K_w$  are bulk modulus of solid and water, respectively,  $\mathbf{k}$  is the absolute permeability of porous media,  $k_{rw}$  and  $k_{rg}$  are water and gas relative permeability,  $\mathbf{L}$  is the differential operator,  $\mathbf{u}$  is displacement,  $\mu_g$  is gas viscosity,  $M_g$  is the molecular weight of gas,  $\theta$  is absolute temperature and  $R$  is universal gas constant.

## 2.3 Constitutive Relations

The systems of equations are nonlinear because the relative permeabilities and the degree of saturation  $S_w$  are a function of capillary pressure which is defined as:  $P_c = P_g - P_w$  and can be determined experimentally or analytically.

Substituting the relation  $\frac{\partial S_w}{\partial t} = \frac{\partial S_w}{\partial P_c} \left( \frac{\partial p_g}{\partial t} - \frac{\partial p_w}{\partial t} \right)$ , equation (4) and (5) are converted to a

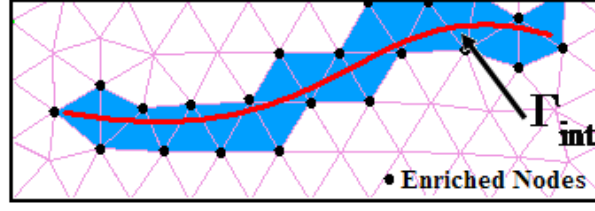
Pressure-Saturation formulation. The constraint  $S_w + S_g = 1$  is considered. There are a variety of relationships that could be used for the relative permeability curves and capillary pressure calculation. In this study the Van Genuchten equation [10] and the Brook and Corey's relationship [2] are used.

## 3 LEVEL SET METHOD

The level set function is defined as a signed distance function  $\Phi$  and given by:

$$\Phi(x) = \begin{cases} \min_{x_i \in \Gamma} \|x - x_i\|, & x \in \Omega_1 \\ -\min_{x_i \in \Gamma} \|x - x_i\|, & x \in \Omega_2 \end{cases} \quad \Phi(x) \in R, x \in \Omega \quad (6)$$

In each time step, the level-set values are advected with the velocity field of the fluid, as



**Figure 1:** A typical finite element mesh with interface, showing enriched nodes and a front

$$\frac{\partial \Phi}{\partial t} + \mathbf{u} \cdot \nabla \Phi = 0 \quad (7)$$

An important issue in the level-set method is the reinitialization of the level-set function in each time step. This is necessary because otherwise the distance property of the level-set function is no longer maintained after the transport. The reinitialization procedure used in [7] is used in this study.

The SUPG (Streamline Upwind Petrov-Galerkin) method is used in this study to stabilize the level set equation. The numerical scheme is stabilized by adding a perturbation to the weighting functions. This perturbation is proportional to the gradient of the standard interpolation functions as:

$$\tilde{N} = N + \tau \mathbf{u} \cdot \nabla N \quad (8)$$

where  $N$  is standard shape function,  $\mathbf{u}$  is flow velocity vector,  $\tau$  is a perturbation parameter and  $\tilde{N}$  is the perturbed shape function. The perturbation parameter presented in [11] is used in this study.

#### 4 EXTENDED FINITE ELEMENT

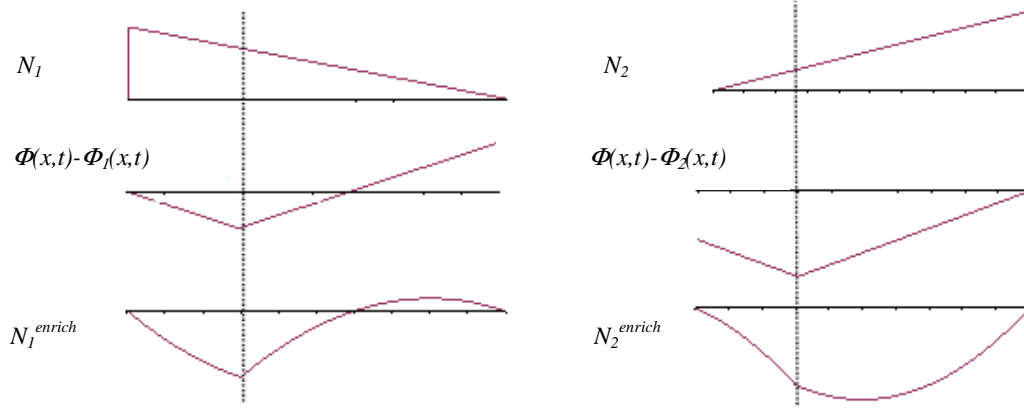
Due to the high gradient in the saturation field at the front between different phases, the XFEM is utilized. The Level set function is utilized for enhancing the element. The enriched approximation is given by:

$$S^h(x, t) = \sum_{I \in N} N_I(x) S_I(t) + \sum_{J \in N_{enriched}} N_J^{enriched}(x, t) a_J \quad (9)$$

where  $S(t)$  are the nodal saturations for the standard finite element and  $a_j(t)$  are additional nodal parameters at the enriched node  $J$ . In constructing the finite element approximation, we distinguish nodes whose element is intersected by the interface  $\Gamma_{int}$  from all others; this set of nodes is indicated by  $N^{enrich}$ . Fig. 1 illustrates which nodes are enriched for a typical example. So for the enriched node an extra degree of freedom should be defined. The enriched shape functions can be defined based on a standard shape function and the level set function as:

$$N_J^{enriched}(x, t) = N_J(x) \left( \left| \Phi^h(x, t) \right| - \left| \Phi^h(x_J, t) \right| \right) \quad (10)$$

Figure (2) shows the enrichment functions for a linear two-node element. The enrichment function is calculated by multiplying a standard shape function to a level set function as represented in equation (10).



**Figure 2:** Example of two enriched finite element shape functions for a linear two node element.

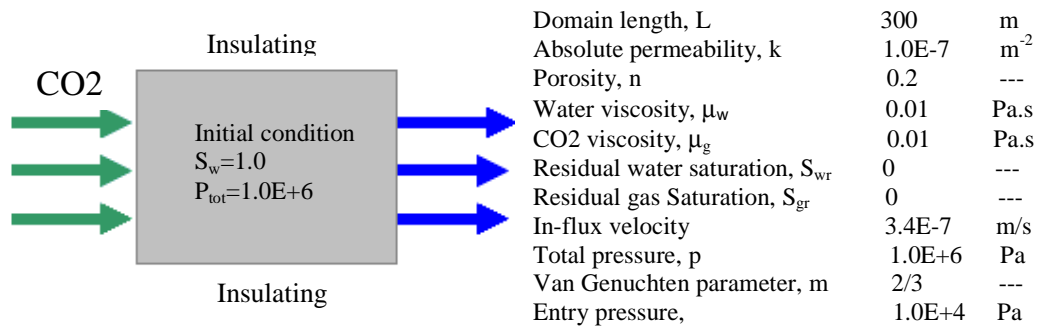
## 5 NUMERICAL EXAMPLE

### 5.1 Buckley-Leverett

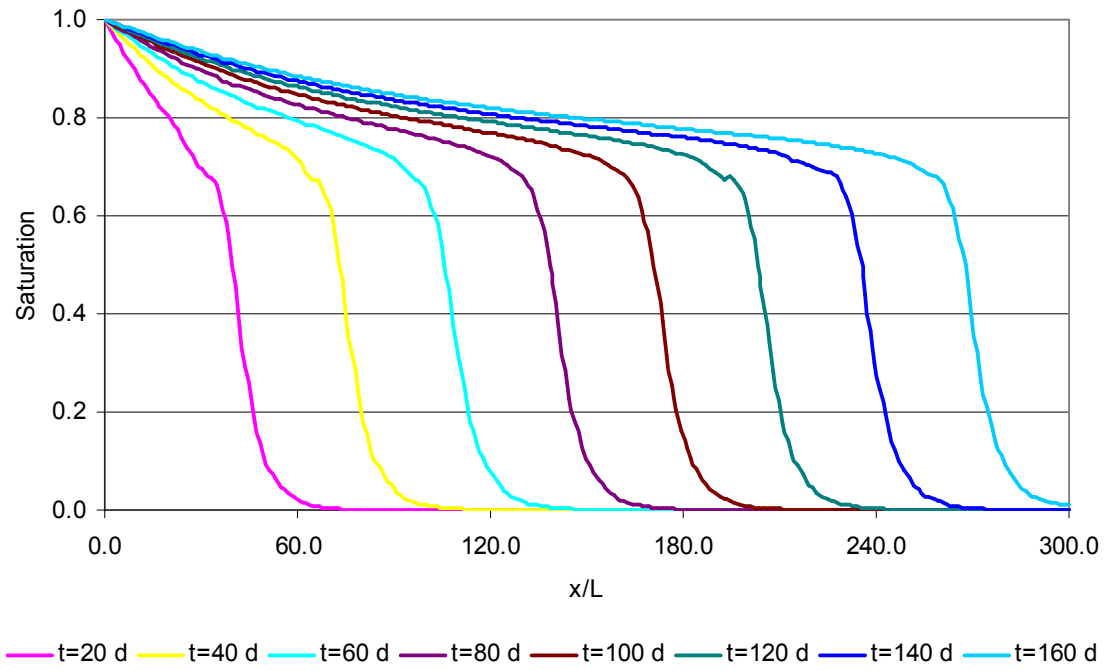
Here, we consider idealized immiscible two phase displacement flow (Buckley-Leverett) in a porous medium. The Buckley-Leverett problem is a simple test problem that has an analytical solution. This problem describes two-phase flow of two immiscible and incompressible fluids in a porous medium, where capillary pressure, gravitational force and deformation are ignored.

The geometry, initial and boundary conditions and model parameter are given in Fig. 3. The CO<sub>2</sub> displaces a wetting phase from left to right. Fig. 4 shows the motion of the saturation front in time. In Fig. 5, the numerical solutions obtained by the standard Galerkin method with 125 elements, by adding artificial diffusion and the PUM method using a coarse mesh with 25 elements are compared with the analytical solution of the Buckley-Leverett problem.

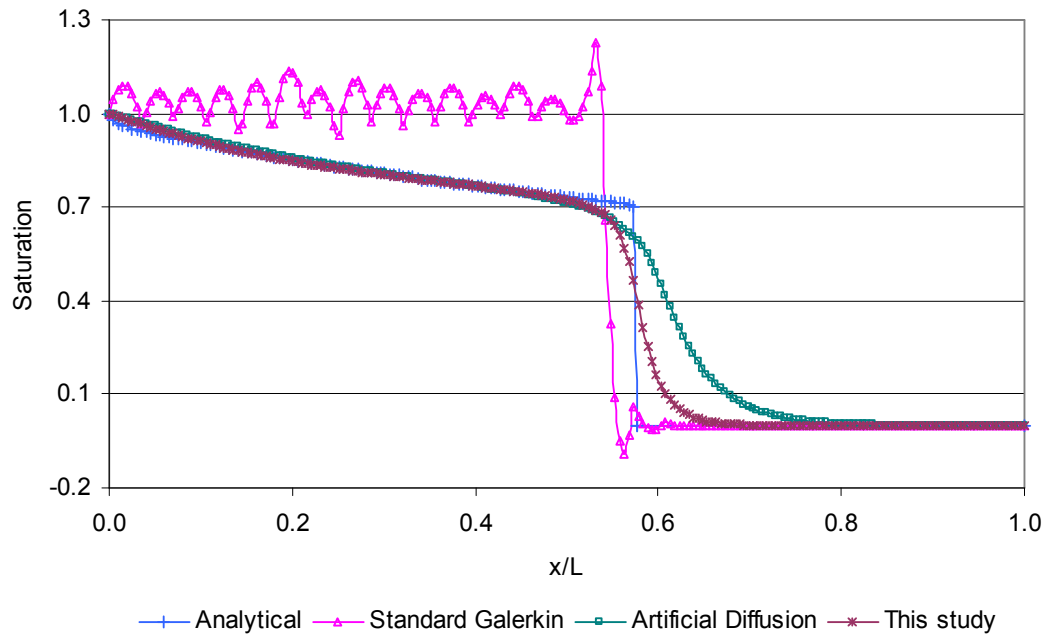
Obviously, the standard Galerkin method exhibits spurious oscillations due to convection. The Petrov Galerkin is highly dissipative, and produces unrealistically smeared sharp fronts. The proposed model produces a sharper front and a smoother profile.



**Figure 3.** The 2D geometry, initial and boundary conditions used for solving the two-phase flow formulation.



**Figure 4.** Saturation profiles for different time using the new approach



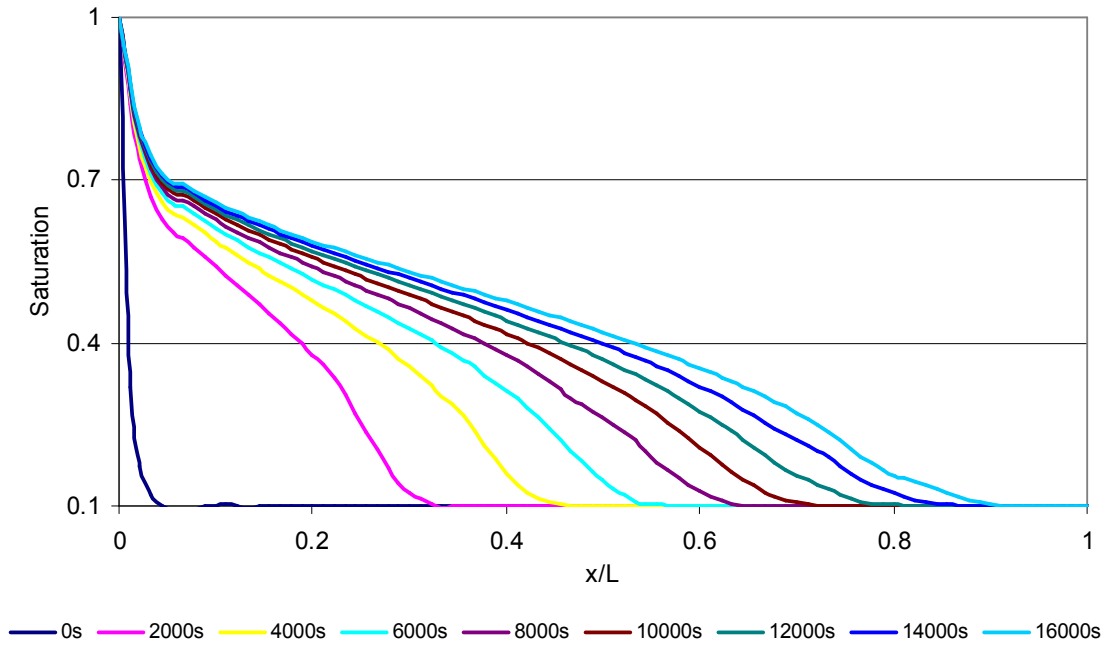
**Figure 5.** Comparison of saturation profile at  $t=100$  day for the different methods.

## 5.2 McWhorter Benchmark Problems

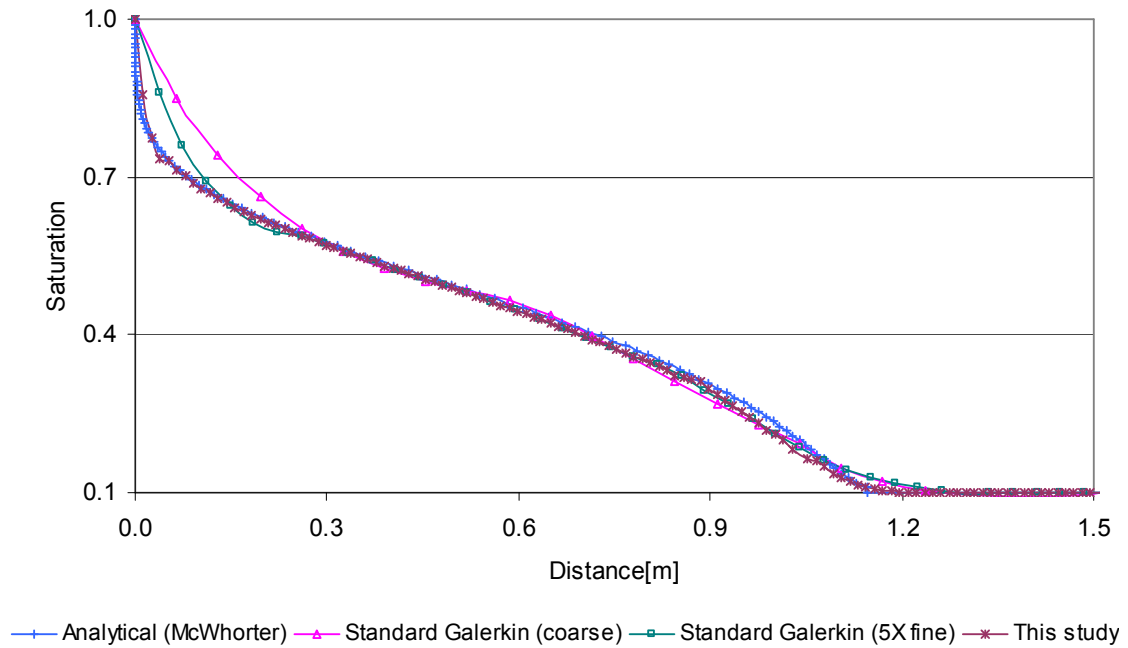
In this test, the capillary pressure is taken into consideration. The deformation of porous media is ignored. The initial saturation of CO<sub>2</sub> is assumed to be 0.1 in the medium. The domain length is assumed to be 2.6 meter. Here, flow is governed by a capillary force when CO<sub>2</sub> saturation at the left end of the medium is kept to one and the CO<sub>2</sub> injection pressure is assumed to be  $2.0 \times 10^5$  Pa.

Figure (6) shows the motion of the saturation front. In Figure 7, numerical results obtained from standard Galerkin method using coarse ( $\Delta x=0.1$ ) and fine meshes ( $\Delta x=0.02$ ) are compared with those obtained from this study and from the semi-analytical solution of the McWhorter problem. The saturation profile at time  $t = 4000$ s for the three methods and the analytical solution is plotted in this figure.

In this case, unlike the Buckley-Leverett problem, which is a convection dominant problem, the capillary effect inserts a diffusive effect in the solution; therefore the front between the two phases is no longer sharp. Consequently the difference between the standard procedure and the extended procedure is less pronounced for this case.



**Figure 6.** Saturation profiles for different times.



**Figure 7.** Comparison of saturation profile at  $t=4000s$  for the different methods.

## 6 CONCLUSIONS

A computational model based on the level set method and the extended finite element method has been developed and utilized for modeling two-phase fluid flow in unsaturated porous media. The level set method is used to track the interface between the injected gas and the resident fluid. A streamline upwind Petrov-Galerkin procedure is used to stabilize the level set equation. The high gradient in the saturation profile is modeled by decomposing it into continuous and discontinuous parts based on the partition of unity method. This makes the model capable of capturing the discontinuity or high gradient in the saturation and stabilizes the result using relatively coarse meshes.

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